At fixed $q^0(0)$ and y_{max}, an increase in $\max_y q^0(y)$ leads to a sharp increase in the rate of fracture. This is illustrated by the data in Fig. 7, where the first column shows the number of cycles N at which the specified probability P(N) is attained for the base variant. The second and third columns show the same for the base variant with $\max_y q^0(y) = 19.61$ MPa and $\max_y q^0(y) = 49.03$ MPa, respectively. It should be noted that with a further increase in $\max_y q^0(y)$, the probability P(N) is determined with a high degree of accuracy only by the indicated maximum.

The results described above qualitatively – and in some cases quantitatively – agree with experimental data. A direct comparison is generally difficult because of the paucity of literature data showing the results of contact fatigue tests and the corresponding initial characteristics of the model examined here.

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BUCKLING OF A NONLINEARLY ELASTIC SLAB LYING ON THE SURFACE OF A LIQUID WITH ALLOWANCE FOR PHASE TRANSFORMATION

V. A. Eremeev

UDC 539.3

The equations of the three-dimensional nonlinear theory of elasticity [1] are used to formulate equilibrium conditions with finite strains for an arbitrary thermoelastic body undergoing a phase transformation. These conditions are then used to study the equilibrium of a circular uniform slab lying on the surface of a melt in a gravitational field. We use the model of a non-Hookian material as the governing relation for the material of the slab, this model being one possible generalization of the model of an incompressible linearly elastic body to the case of finite strains. The method of superimposing a small strain on a finite strain [1] is used to study local loss of stability of the slab due to its compression in the radial direction. The critical strains are determined numerically. A similar approach is used to study buckling of the slab in the absence of phase transformation.

1. We will examine the equilibrium of a thermoelastic body undergoing a first-order liquid–solid phase transformation. Similar transitions were studied within the framework of continuum mechanics in [2-7], where various approaches were employed to obtain relations describing the phase transition at the phase boundary. A characteristic feature of the problem of the equilibrium of a thermoelastic body under phase-transformation conditions is the presence of an a priori unknown phase boundary. As an auxiliary phase-transformation condition serving to determine the position of the phase boundary, we choose the equation of the fusion curve [8]. This equation expresses the dependence of the melting point on pressure in the liquid [8].

Let the volume occupied by the body in the reference configuration be equal to v. We represent the external boundary of the body as the union of the surface γ separating the body from the liquid and the surface $\sigma = \sigma_1 \cup \sigma_2 \cup \sigma_3 = \sigma_4 \cup \sigma_5$ (Fig. 1). The body is de-

Rostov-on-Don. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 141-147, May-June, 1991. Original article submitted July 12, 1989; revision submitted January 22, 1990.

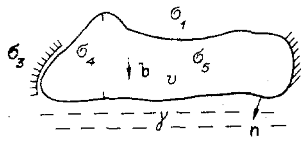

Fig. 1

formed by body forces $\mathbf{b}$ and by loads $\mathbf{d}$ distributed over the surface σ_1. The displacements are assigned on the surface σ_2. Part of the boundary σ_3 is in contact with a smooth rigid surface. The temperature Θ_0 is assigned on the surface σ_4, while the heat flux s is assigned on σ_5.

The equilibrium and heat-balance equations and the boundary conditions on the surface σ have the following form [1] for a nonlinearly thermoelastic body

$$\begin{gathered}\nabla^0\cdot\mathbf{D}+\rho_0\mathbf{b}=0,\ \nabla^0\cdot\mathbf{h}^0=0,\\ \mathbf{n}\cdot\mathbf{D}|_{\sigma_1}=\mathbf{d},\quad \mathbf{R}|_{\sigma_2}=\mathbf{R}_0,\quad \mathbf{n}\cdot\mathbf{D}\cdot(\mathbf{E}-\mathbf{NN})|_{\sigma_3}=0,\\ \mathbf{n}\cdot(\mathbf{R}-\mathbf{r})|_{\sigma_3}=0,\quad \Theta|_{\sigma_4}=\Theta_0,\quad \mathbf{n}\cdot\mathbf{h}^0|_{\sigma_5}=s,\end{gathered}\tag{1.1}$$

where ∇^0 is the gradient operator in the reference configuration; $\mathbf{D}$ is the Piola stress tensor; $\mathbf{h}^0$ is the Piola heat-flux vector; ρ_0 is the density of the material in the reference configuration; $\mathbf{r}$ and $\mathbf{R}=\mathbf{R}(\mathbf{r})$ are the position vectors of a point of the body in the initial and deformed states; $\mathbf{E}$ is the unit tensor; $\mathbf{n}$ is a unit normal to the surface σ; $\mathbf{N}$ is a unit normal to the boundary of the body in the deformed state; Θ is temperature. The pressure in the liquid p and its temperature can be determined from the equilibrium and heat-balance equations, written in Eulerian coordinates [1, 9]:

$$-\nabla p+\rho_-\mathbf{b}=0,\ \nabla\cdot\mathbf{h}=0,\ \mathbf{h}=J^{-1}\mathbf{C}^T\cdot\mathbf{h}^0,\ J=\det\mathbf{C}.\tag{1.2}$$

Here ρ_- is the density of the liquid; ∇ is the gradient operator in Eulerian coordinates, connected with ∇^0 by the formula $\nabla=\mathbf{C}^{-1}\cdot\nabla^0$; $\mathbf{h}$ is the heat-flux vector; $\mathbf{C}=\nabla^0\mathbf{R}$ is the strain gradient. The boundary conditions for the liquid on surfaces different than γ are written in standard form [9] and are not further discussed.

At the phase boundary γ we require satisfaction of compatibility conditions for the temperature, stress, and heat-flux vectors in the solid and the liquid:

$$\mathbf{n}\cdot[\mathbf{D}]=0,\ [\Theta]=0,\ \mathbf{n}\cdot[\mathbf{h}^0]=0.\tag{1.3}$$

The brackets in (1.3) denote a sudden change in the corresponding quantity with the crossing of γ. The boundary condition for the stresses can be changed to a form corresponding to the action of hydrostatic pressure p on the solid from the direction of the liquid [1]:

$$\mathbf{n}\cdot\mathbf{D}=-pJ\mathbf{C}^{-1}\cdot\mathbf{n}.\tag{1.4}$$

The condition of phase equilibrium in the equation of the fusion curve has the form [8]

$$\Theta|_\gamma=\Theta^*(p)\tag{1.5}$$

[$\Theta^*(p)$ is a known function].

Let us examine a special case described by the above equations. Let the boundary conditions be such that a uniform temperature field is realized in the solid: $\Theta=\Theta^0$. If the body is acted upon by body forces, then the pressure in the liquid will depend on the coordinates. Thus, with equilibrium of a homogeneous incompressible fluid in a gravitational field, pressure depends linearly on the vertical coordinate. In this case, the position of the phase boundary can be found from a condition obtained by transformation of Eq. (1.5):

$$p|_\gamma=p^*(\Theta^0).\tag{1.6}$$

Here, $p^*(\Theta)$ is the inverse of $\Theta^*(p)$. It follows from Eq. (1.6) that pressure is constant on the phase boundary.

2. Let us make use of Eqs. (1.1), (1.2), (1.4), and (1.6) to study the equilibrium and stability of a heavy circular slab located in a uniform temperature field. The stability of plates was studied in [10-13] on the basis of the three-dimensional nonlinear theory of elasticity. In the ice-flow theory constructed in [14], the possibility of melting of the ice was accounted for along with the other factors discussed above.

We will assume that the elastic slab lies on a layer of a homogeneous incompressible fluid whose depth is assumed to be infinite. Radial displacements are assigned on the lateral surface of the slab and shear stresses are absent. Similar boundary conditions were examined in [10-13]. The top surface of the slab is not loaded, while in accordance with (1.4) the bottom surface is loaded by hydrostatic pressure which comes from the direction of the liquid and balances the weight of the slab. Similar boundary conditions can be realized by placing the slab in a rigid smooth cylindrical yoke allowing displacement of the slab in the vertical direction and permitting a change in its radius. In the reference configuration, the slab occupies the volume $0 \le r \le a$, $-h \le z \le 0$. Here and below, r, z, and φ are Lagrangian cylindrical coordinates, a is the radius of the slab, and h is its thickness. In the case of phase transformation, the thickness of the slab is determined by means of phase equilibrium condition (1.6).

The constitutive law of the elastic body will be described using the model of a non-Hookian material [1] $\mathbf{D} = 2\mu\mathbf{C} - q\mathbf{C}^{-T}$ (μ is the elastic constant). The strain-independent function q arises as a result of the incompressibility condition $\det \mathbf{C} = 1$, similarly to the pressure function in the statics of incompressible fluids. Being an independent characteristic of the stress state, this function is subject to determination together with the strain field. In the case of small strains, a non-Hookian material obeys Hooke's law with the shear modulus μ and Poisson's ratio 1/2.

The pressure in a homogeneous incompressible medium located in a gravitational field has the form [9]

$$p = p_0 - \rho_- g Z \qquad (2.1)$$

(p_0 is a constant of integration; g is acceleration due to gravity). Here and below, R, Z, and Φ are Eulerian cylindrical coordinates. Since pressure at the phase boundary is constant in accordance with (1.6), it follows from (2.1) that in a gravitational field the phase boundary in the deformed state can be a horizontal plane

$$Z = \text{const.} \qquad (2.2)$$

We will examine the stress-strain state of a slab in which this state is plane. In this case, the following formulas [12, 13] give a satisfactory incompressibility condition

$$R = \lambda r, \ \Phi = \varphi, \ Z = \lambda^{-2} z \qquad (2.3)$$

(λ is a strain parameter characterizing the compression of the slab in the radial direction). It is easily shown that transformation (2.3) satisfies the boundary conditions on the lateral surface of the slab, while the equilibrium equations and boundary conditions on the ends of the slab reduce to the form

$$\frac{d}{dz} D_{zz} - \rho_0 g = 0, \quad D_{zz}|_{z=0} = 0, \quad D_{zz}|_{z=-h} = -p^*\lambda^2,$$

where $D_{zz} = -q\lambda^2 + 2\mu\lambda^{-2}$. We find from this equation that

$$D_{zz} = \rho_0 g z, \ q \equiv q^0(z) = 2\mu\lambda^{-4} - \rho_0 g \lambda^{-2} z, \ h = p^*\lambda^2/(\rho_0 g). \qquad (2.4)$$

The last relation in (2.4), connecting the thickness of the slab h with the pressure at the phase boundary p^*, is the condition of equilibrium of a slab lying on a liquid surface and can be obtained directly by analyzing the equations of hydrostatics [9]. In contrast to problems concerning the equilibrium of floating bodies, in the present case the pressure acting on the phase boundary is given and the thickness of the slab is determined.

3. The local stability of the equilibrium of a slab in a plane state will be studied by the static method. This method entails finding equilibrium positions which differ little from the prescribed position and determining the critical values of the strain parameter λ at which the linearized equilibrium equations and boundary conditions can have nontrivial solutions.

The axisymmetric deformation of the slab, describing its deviation from the plane state, is written in the form [13]

$$R = \lambda r + u(r, z),\ \Phi = \varphi,\ Z = \lambda^{-2}z + v(r, z). \tag{3.1}$$

Written in Lagrangian coordinates so as to account for its change as a result of phase transformation, the equation of the phase boundary also differs from the equation of a plane

$$z = -h + \zeta(r). \tag{3.2}$$

The function $\zeta(r)$ describes the change in the thickness of the slab caused by its melting or by crystallization of the liquid. Inserting (3.2) into the last relation of (3.1), we obtain a relation which corresponds to (2.2) and connects the vertical displacement of points of the bottom surface of the slab v with the function ζ:

$$Z|_{z=-h+\zeta(r)} \equiv v(r, -h + \zeta(r)) - \lambda^{-2}h + \lambda^{-2}\zeta(r) = \text{const}. \tag{3.3}$$

The linearized equations of equilibrium and compressibility and the boundary conditions on the surface not in contact with the liquid are written in the form [1, 12, 13]

$$\begin{gathered}\nabla^0\cdot\mathbf{D}^\cdot = 0,\ \mathbf{C}^{-T}\cdot\cdot\nabla^0\mathbf{w} = 0,\ \mathbf{w} = u\mathbf{e}_r + v\mathbf{e}_z,\\ \mathbf{e}_z\cdot\mathbf{D}^\cdot|_{z=0} = 0,\ \mathbf{e}_r\cdot\mathbf{D}^\cdot\cdot\mathbf{e}_z|_{r=a} = 0,\ \mathbf{e}_r\cdot\mathbf{D}^\cdot\cdot\mathbf{e}_\varphi|_{r=a} = 0,\\ u(a, z) = 0,\ \mathbf{D}^\cdot = 2\mu\nabla^0\mathbf{w} - q^\cdot\mathbf{C}^{-T} + q\mathbf{C}^{-T}\cdot\nabla^0\mathbf{w}^T\cdot\mathbf{C}^{-T}.\end{gathered} \tag{3.4}$$

Here, **w** is the vector of the small additional displacements; $\mathbf{D}^\cdot$ is the linearized Piola stress tensor, describing the change in the stress state with the superposition of the additional displacements; $q^\cdot$ is a small perturbation of the function q; **C** is the strain gradient in the main stress state, determined by Eqs. (2.3); $\mathbf{e}_r, \mathbf{e}_z, \mathbf{e}_\varphi$ is an orthonormalized basis connected with the Lagrangian cylindrical coordinates. The functions u, v, and $q^\cdot$ completely characterize the axisymmetric stress–strain state in the body of slab, this state differing little from initial equilibrium strain state.

To obtain linearized equations for the phase boundary, we examine nonlinear boundary-value problems (1.4), (3.3) with allowance for Eqs. (3.1), (3.2). The normal vector to the curved phase boundary determined by Eq. (3.2) is given by the relation

$$\mathbf{n} = \left(\left(\frac{\partial\zeta}{\partial r}\right)^2 + 1\right)^{-1/2}\left(-\frac{\partial\zeta}{\partial r}\mathbf{e}_r + \mathbf{e}_z\right). \tag{3.5}$$

Using (3.5), at $z = -h + \zeta(r)$ we can change condition (1.4) to the form

$$\mathbf{e}_z\cdot\mathbf{D} - \frac{\partial\zeta}{\partial r}\mathbf{e}_r\cdot\mathbf{D} = -p^*\left(\mathbf{e}_z\cdot\mathbf{C}^{-T} - \frac{\partial\zeta}{\partial r}\mathbf{e}_r\cdot\mathbf{C}^{-T}\right), \tag{3.6}$$

where

$$\mathbf{e}_z\cdot\mathbf{D} = \left[2\mu\left(\lambda^{-2} + \frac{\partial v}{\partial z}\right) - q\left(\lambda + \frac{u}{r}\right)\left(\lambda + \frac{\partial u}{\partial r}\right)\right]\mathbf{e}_z + \left[2\mu\frac{\partial u}{\partial z} + q\left(\lambda + \frac{u}{r}\right)\frac{\partial v}{\partial r}\right]\mathbf{e}_r,\ \mathbf{e}_r\cdot\mathbf{D} = \left[2\mu\left(\lambda + \frac{\partial u}{\partial r}\right) -\right.$$

$$\left. - q\left(\lambda + \frac{u}{r}\right)\left(\lambda^{-2} + \frac{\partial v}{\partial z}\right)\right]\mathbf{e}_r + \left[2\mu\frac{\partial v}{\partial r} + q\left(\lambda + \frac{u}{r}\right)\frac{\partial u}{\partial z}\right]\mathbf{e}_z,$$

$$\mathbf{e}_z\cdot C^{-T} = \left(\lambda + \frac{u}{r}\right)\left[\left(\lambda + \frac{\partial u}{\partial r}\right)\mathbf{e}_z - \frac{\partial v}{\partial r}\mathbf{e}_r\right],\quad \mathbf{e}_r\cdot\mathbf{C}^{-T} = \left(\lambda + \frac{u}{r}\right)\left[\left(\lambda^{-2} + \frac{\partial v}{\partial z}\right)\mathbf{e}_r - \frac{\partial u}{\partial z}\mathbf{e}_z\right],\quad q = q^0(z) + q^\cdot(r, z).$$

In the case of the absence of a phase transformation ($\zeta = 0$), nonlinear relations (3.6) take the form shown in [13]. If we keep no terms higher than the first degree for the unknown functions u, v, $q^\cdot$, and ζ in (3.6) and we take (2.4) into account, we find linearized boundary conditions at $z = -h$

$$\begin{gathered}\partial v/\partial z - q^\cdot\lambda^2/2\mu - \lambda^{-2}(\partial u/\partial r + u/r) + \rho_0 g\zeta/2\mu = 0,\\ \partial u/\partial z + \lambda^{-3}\partial v/\partial r - (\lambda - \lambda^{-5})\partial\zeta/\partial r = 0.\end{gathered} \tag{3.7}$$

TABLE 1

$\bar{h}$	β							
	0,01		0,05		0,1		0,2	
	λ′	λ*	λ′	λ*	λ′	λ*	λ′	λ*
0,1	0,9907	0,9813	0,9864	0,9733	0,9812	0,9641	0,9718	0,9481
0,2	0,9664	0,9304	0,9643	0,9267	0,9616	0,9224	0,9567	0,9142
0,3	0,9240	0,8368	0,9227	0,8349	0,9210	0,8326	0,9178	0,8281

TABLE 2

$\bar{h}$	$\lambda^*\cdot 10^4$	$\lambda'\cdot 10^4$	n	$\bar{h}$	$\lambda^*\cdot 10^4$	$\lambda'\cdot 10^4$	n
0,005	9915	9957	12	0,04	9760	9876	2
0,01	9881	9939	7	0,06	9699	9847	2
0,02	9833	9914	4	0,08	9654	9817	1

The unknown perturbation of the phase boundary ζ can be determined by linearizing Eq. (3.3)

$$\lambda^2 v(r, -h) + \zeta(r) = 0. \tag{3.8}$$

Equation (3.8) makes it possible to exclude the unknown ζ from boundary conditions (3.7), leading to the relations

$$\begin{gathered} \partial v/\partial z - q^{\bullet}\lambda^2/2\mu - \lambda^{-2}(\partial u/\partial r + u/r) - \rho_0 g\lambda^2 v/2\mu = 0, \\ \partial u/\partial z + \lambda^3 \partial v/\partial r = 0. \end{gathered} \tag{3.9}$$

We also obtain linearized boundary conditions for the bottom surface of the slab when there is no phase transformation. In this case, the nonlinear boundary condition corresponding to (1.4) and assigned at z = −h has the form

$$\mathbf{e}_z\cdot\mathbf{D} = -p\mathbf{e}_z\cdot\mathbf{C}^{-T}, \tag{3.10}$$

where the pressure p is given by Eqs. (2.1), (3.1) and the remaining expressions are determined as they were in (3.6). Linearization of (3.10) leads to the equations

$$2\partial v/\partial z - \lambda^2 q^{\cdot}/2\mu - \rho_- g\lambda^2 v/2\mu = 0,\ \partial u/\partial z + \lambda^{-3}\partial v/\partial r = 0. \tag{3.11}$$

Following [12, 13], we seek the solution of Eqs. (3.4) in the form

$$\begin{gathered} u(r, z) = \sum_{n=1}^{\infty} U_n(z) J_1(\gamma_n r/a), \quad v(r, z) = V_0(z) + \sum_{n=1}^{\infty} V_n(z) J_0(\gamma_n r/a), \\ q^{\cdot}(r, z) = Q_0(z) = \sum_{n=1}^{\infty} Q_n(z) J_0(\gamma_n r/a) \end{gathered} \tag{3.12}$$

(γ_n are roots of the Bessel function J_1). The functions $V_0(z)$, $Q_0(z)$ correspond to small strains of the slab which are independent of the radial coordinate. Having chosen such a solution, we find that the boundary conditions on the lateral surface are satisfied identically. As in [10, 12, 13], we will be interested mainly in modes of instability accompanied by bending of the slab. Insertion of (3.12) into (3.4), (3.9) and (3.4), (3.11) leads to a linear homogeneous boundary-value problem for a system of ordinary differential equations relative to the functions $V_0(z)$, $Q_0(z)$, $U_n(z)$, $V_n(z)$, $Q_n(z)$. The general solution of this system is given by the formulas

$$\begin{gathered} V_n(z) = C_1 \operatorname{ch} a_n z + C_2 \operatorname{sh} a_n z + C_3 \operatorname{ch}\frac{a_n z}{\lambda^3} + C_4 \operatorname{sh}\frac{a_n z}{\lambda^3}, \\ U_n(z) = -\frac{\lambda^3}{a_n}\frac{d}{dz}V_n(z),\ Q_n(z) = -\alpha C_1 \operatorname{ch} a_n z - \alpha C_2 \operatorname{sh} a_n z + \end{gathered}$$

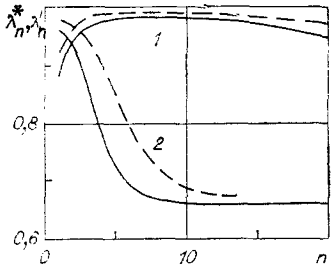

Fig. 2

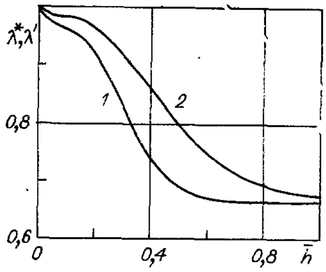

Fig. 3

$$+ C_3\left(a_n \lambda(\lambda^{-6} - 1)\,\mathrm{sh}\frac{a_n z}{\lambda^3} - \alpha\,\mathrm{ch}\frac{a_n z}{\lambda^3}\right) + C_4\left(a_n\lambda(\lambda^{-6} - 1)\,\mathrm{ch}\frac{a_n z}{\lambda^3} - \alpha\,\mathrm{sh}\frac{a_n z}{\lambda^3}\right),$$

$$V_0(z) = C_0', \quad Q_0(z) = C_0'', \quad a_n = \gamma_n/a, \quad \alpha = \rho_0 g/2\mu.$$

Inserting the resulting solution into the boundary conditions with $z = 0$, $z = -h$, we arrive at a homogeneous system of linear algebraic equations for determination of the constants of integration C_0', C_0'', C_k ($k = 1, \ldots, 4$). It can be shown that C_0', C_0'' are equal to zero.

Critical values of the strain parameter λ_n^* obtained with allowance for phase transformation were determined numerically from the condition that the system of equations for C_k have a nontrivial solution. We also calculated critical values of the strain parameter λ_n' in the absence of phase transformation. The largest values of λ_n^*, λ_n' ($n = 1, 2, \ldots$) will be designated as λ^*, λ'. These maxima correspond to the smallest forces that can be applied to the lateral surface of the slab and still cause buckling to occur. The calculations showed that it is always the case that $\lambda^* < \lambda'$. This indicates that allowance for phase transformation in the given problem leads to an increase in the critical loads and strains at which the slab becomes unstable, i.e., phase transformations have a stabilizing effect. Tables 1 and 2 show values of λ^* and λ' for different relative thicknesses $\bar{h} = h/a$ and different values of the parameter $\beta = \rho_0 ga/2\mu$, characterizing the effect of gravity. The form of the slab after loss of stability is determined by the number n. The number of the buckling mode which corresponds to the values of λ^*, λ' shown in Table 1 is $n = 1$. For sufficiently thin slabs, the lowest critical loads correspond to a buckling-mode number $n > 1$. This distinguishes the present problem, accounting for the effect of gravity and the liquid, from the results obtained in [10, 12, 13] (where n was always equal to unity). Table 2 shows values of λ^* and λ' and the corresponding numbers n with $\beta = 0.1$. Figure 2 shows the dependence of λ_n^* and λ_n' on n at $\beta = 0.1$ and $\bar{h} = 0.01$ and 0.1 (curves 1, 2). The solid lines show the dependence of λ^* on n, while the dashed lines show the dependence of λ' on n. Figure 3 shows the dependence of λ^* and λ' on thickness with $\beta = 0.1$ (lines 1 and 2).

We thank L. M. Zubov for his help in conducting this investigation.

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BEGINNING OF PLASTIC YIELDING IN A STRESS CONCENTRATION ZONE

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UDC 539.214;539.374;539.375

Classical strength criteria are currently being widely used in the strength design of structural elements. Here, it is assumed that plastic yielding begins when, in accordance with the chosen criterion, the limiting stress state is attained at even one (the most heavily stressed) point of the structure. However, these criteria do not always consider how the beginning of plastic flow is affected by the nonuniformity of the stress distribution near the point of greatest stress.

The subject of the effect of nonuniformity of the stress state on the yield point in the region where the stresses are maximal has long been of interest to researchers [1-3]. Subsequent to [1-3], investigators made use of the gradient approach proposed in [1] to evaluate this nonuniformity and its effect on the local yield point at the most heavily stressed point of the body [4-6]. Signs of plastic flow in the region of maximum stresses were considered to be the appearance of Lüders' lines in specimens of mild steel [1] and deviations from elastic strain laws detected by strain gauges or other means [4, 5]. It was noted that these indications of plastic yielding are manifest when the stresses at the most heavily stressed point exceed the yield point in a uniform stress state σ_y. Recent experiments have detected deviations from elastic strain laws by the highly sensitive method of holographic interferometry [7, 8]. These experiments have also confirmed that there is an increase in the local yield point at the most heavily stressed point of the body. The results that were obtained were used as a basis for proposing a gradient criterion for the onset of plastic flow in a nonuniform stress state [9-11].

In the present study, we use the example of the tension of a plate with an elliptical hole to examine the range of validity of the gradient criterion and the continuum model in the case of very small holes. We note that there is a connection between this criterion and the structure of the material, and we show that the criterion actually reflects the energy dependence of the beginning of plastic flow for a fairly broad range of stress-concentration factors and hole sizes.

<u>1. Range of Validity of the Gradient Criterion and the Continuum Model in the Case of Very Small Holes.</u> In accordance with the gradient criterion, in a nonuniform stress state plastic strains occur only when an equivalent stress – let this be the stress intensity σ_i – at the most heavily stressed point of the given body $\sigma_i^{\max}$ exceeds σ_y and reaches the local yield point σ_y^{ℓ}:

$$\sigma_y^{\ell} = \sigma_y\left(1 + \sqrt{L_0 G/\sigma_i^{\max}}\right). \quad (1.1)$$

Here, G = $|\mathrm{grad}\ \sigma_i|$ is the modulus of the gradient of σ_i at the point subject to the greatest

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 147-152, May-June, 1991. Original article submitted January 9, 1990.

